Confidence Intervals

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Today's Blueprint

Last Class

- Hypothesis Testing
  - The general idea
  - Error in Hypothesis Testing

Today's Class

- Confidence Intervals
  - The General Idea
  - Error in Confidence Intervals

The Big Picture…

The Big Picture

- *Inferential statistics* = Using a sample to learn about a population
- Two ways to learn about a population:
  - Confidence intervals (inductive approach)
  - Hypothesis testing (deductive approach)

The Big Picture

- Hypothesis testing (deductive approach)
  - Allows us to use sample data to *test a claim* about a population

The Big Picture

- Confidence intervals: (inductive approach)
  - Allow us to use sample data to *estimate* a population value
  - …here's how…

The General Idea of Confidence Intervals

The General Idea

- Allow us to use sample data to *estimate* a population value, like the true mean or the true proportion
- …here's how…

The General Idea

- Another term for a statistic is a *point estimate*, since we are estimating the parameter value (using statistics)

The General Idea

- Because of *sampling error*, we know that it's not likely that our sample statistic will be equal to the population parameter

The General Idea

- Instead, our sample statistic will fall into an range (i.e. *interval*) of values
- We will have to be satisfied knowing that the statistic is "close to" the parameter

The General Idea

- That leads to the obvious question:
  - *What is "close"?*

The General Idea

- In other words, how *confident* can we be that the value of the statistic falls within a certain “distance” of the parameter?

The General Idea

- Put differently, what is the *probability* that the parameter’s value is within a certain range of the statistic’s value?
- This range is called the *confidence interval*

Insert Umpire Analogy Here

The General Idea

- The *confidence level* is the *probability* that the value of the parameter falls within the range specified by the *confidence interval* surrounding the statistic

The General Idea

- In this case:
  - Sample = Pitches
  - True parameter = Center of the strike zone
  - Interval = Circle(s) drawn around each pitch

The General Idea

- In this case, the umpire is 95% confident that his circles will capture the center of the strike zone in repeated samples (pitches)

Error in Confidence Intervals

Error in Confidence Intervals

- A 95% *confidence interval* procedure gives us a 0.95 probability of the interval containing the true parameter

Error in Confidence Intervals

- The umpire specified this *interval* before he saw the pitches
  - Confidence intervals are specified before the data are collected
  - Pitches = random variables (outcome is unknown until pitches are thrown)

Error in Confidence Intervals

- Once the data are collected and the interval is computed, the procedure either works or it doesn’t
  - Example: The pitches either do or do not fall within the strike zone 95% of the time

Error in Confidence Intervals

- Therefore:
  - We cannot make meaningful probability statements about the actual interval,
  - We can only comment about the method that we used to construct the interval

Error in Confidence Intervals

- How close to the population parameter do we expect our interval to fall?
  - The interval will capture values above and below the true parameter

Error in Confidence Intervals

- Therefore, our confidence interval provides a plus/minus amount (known as the level of accuracy) which expresses how close we are to the true parameter with a given level of confidence

Error in Confidence Intervals

- This is the logic behind the *margin of error*
  - The *margin or error* tells us how accurate we believe our guess of the true parameter is, based on the variability of the estimate

Insert Normal Curve Example Here

To Recap:

- A *confidence interval* is an interval estimate of an unknown population parameter based on a random sample from the population.

To Recap

- An increase in sample size leads to a decreased interval width
  - large samples have narrower widths than small samples (all other things being equal)

To Recap

- The *level of confidence* tells the probability the method produced an interval that includes the unknown parameter
  - The probability relates to the method (data, interval), not to the parameter

To Recap

- All things being equal, higher confidence levels have wider intervals than lower confidence levels
- Narrow widths and high confidence levels are desirable, but these two things affect each other