

Probability

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Today's Blueprint

Last Class

- Statistics
 - What it "is"
 - What they "are"
 - How to Lie with them
 - How to tell the truth with them

Today's Class

- Probability:
 - The Logic Behind It
 - [Some] Rules to It

The Logic Behind Probability

The Logic Behind It

- A theory that began in 17th century France
 - Refers to the relative likelihood of outcomes occurring

The Logic Behind It

- Basic Definitions
 - Random Variables
 - Random Experiments
 - Elementary Outcomes
 - Sample Spaces

The Logic Behind It

- 1) Random variable:
- A variable whose outcome is uncertain until a random "experiment" is done
 - Example: A coin flip (the flip is the variable)

The Logic Behind It

- 2) Random experiment
- The process of observing a chance event
 - Example: Writing down a series of coin flips

The Logic Behind It

- 3) Elementary outcomes:
- All the possible values a random variable can take in a random experiment
 - Example: A flipped coin can either fall "heads" (H) or "tails" (T)

The Logic Behind It

- 4) Sample Space:
- The set of all elementary outcomes
 - Example: Flipping 2 coin gives you the following sample space: {H,T}.
 - Within that space are the following possible combinations {(H,H) (H,T) (T,H) (T,T)}

The Logic Behind It

- Two approaches to thinking "probabilistically":
 - Relative frequencies approach:

- Subjective approach

The Logic Behind It

1) Relative frequencies approach:

- Useful when you can that we can perform the “experiment” repeatedly under similar conditions
 - Example: What is the likelihood of me making 5 out of 10 free throws?

The Logic Behind It

2) Subjective approach:

- Useful when you can that we can only perform the experiment once
- Assign a value to the event that reflects your likelihood of the event happening
 - Example: What is the likelihood of me getting an A in my class?

The Logic Behind It

- Probability is how we take a sample and test hypotheses about a population
 - Testing the probability that a random variable will take on some elementary outcome

Some Rules of Probability

Some Probability Rules:

- All probabilities are between 0 and 1 inclusive ($0 \leq \text{Pr} \leq 1$)
 - The probability of an event which cannot occur is 0.
 - The probability of an event which must occur is 1.
 - The sum of all the probabilities in the sample space is 1
 - The probability of any event which is not in the sample space is zero.

Some Probability Rules

Based on that logic:

- The Converse Rule
- The Addition Rule
- The Multiplication Rule

Some Probability Rules

- The Converse Rule
- The probability of a specific elementary outcome not occurring is equal to 1 minus the probability that it will occur
- Example:
 - $\text{Pr}(\text{Coin Flip} = \text{Heads}) = \frac{1}{2} = 0.5$
 - $\text{Pr}(\text{Coin Flip} = \text{Tails}) = \frac{1}{2} = 0.5$
 - Therefore: $\text{Pr}(\text{Heads}) = 1 - \text{Pr}(\text{Tails}) = 1.0 - 0.5 = 0.5$

Some Probability Rules

- The Addition Rule
- The Probability of observing any one of several *mutually exclusive* elementary outcomes is equal to the sum of their separate probabilities
- Mutually Exclusive = Independent Outcomes
 - The occurrence of one outcome does not influence the probability of another outcome
 - Example:
 - $\text{Pr}(\text{Card Draw} = \text{Ace of Spades OR Ace of Hearts}) = \frac{1}{52} + \frac{1}{52} = \frac{2}{52} \approx 0.04$

Some Probability Rules

- The Multiplication Rule
- The probability of observing a combination of *independent* outcomes equals the product of their separate probabilities

- Example:
 - $\Pr(\text{Card Draw} = \text{Ace of Spades AND Ace of Hearts}) = 1/52 * 1/52 = 1/2704 \approx .00037$

References

- FYI:
 - Levin, Jack and James Alan Fox. 2003. Elementary Statistics In Social Research, 9th Edition. Boston, MA: Pearson Education Group, Inc.
 - Wonnacott, Thomas H. and Ronald J. Wonnacott. 1990. Introductory Statistics, 5th Edition. New York, NY: John Wiley & Sons.
 - Stats: Introduction to Probability <http://www.richland.cc.il.us/james/lecture/m170/ch05-int.html>